



Gravitational Lensing

Commentary and Discussion

Jiayang Sun

Department of Statistics
Case Western Reserve University
Cleveland, OH 44106

jsun@case.edu



Provide commentaries on
the following two papers:
Chris Kochanek: Turning
AGN Microlensing from a
Curiosity into a Tool
Gary Bernstein: Statistical
Challenges of Weak
Gravitational Lensing

Wonder of Gravitational Lensing



Thanks to Chris and Gary for their papers on gravitational lensing, which vividly refreshed my memory of Pete Kernan's "trick" on messing up someone's picture:



♣ Produced by lensing Gary's picture on

<http://theory2.phys.cwru.edu/~pete/GravitationalLens/GravitationalLens.html>

What's Gravitational Lensing (GL)



**Massive bodies can bend
the path of light rays**

This effect is called GL

**magnifies, distorts w.
obvious traces and
may produce multiple
images**

Strong Lensing

**produces subtle
distortion
(shear, magnification)**

Weak Lensing

What's Gravitational Lensing (GL)



Massive bodies can bend
the path of light rays

This effect is called GL

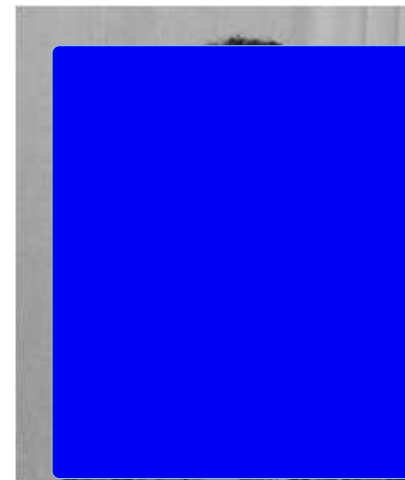
magnifies, distorts w.
obvious traces and
may produce multiple
images

Strong Lensing



produces subtle
distortion
(shear, magnification)

Weak Lensing



What's Gravitational Lensing (GL)



Massive bodies can bend
the path of light rays

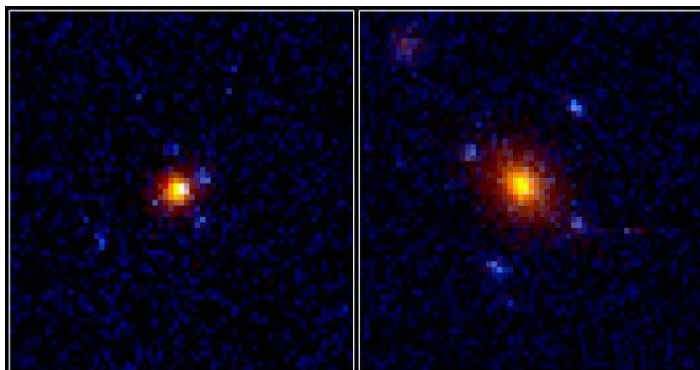
This effect is called GL

magnifies, distorts w.
obvious traces and
may produce multiple
images

Strong Lensing

produces subtle
distortion
(shear, magnification)

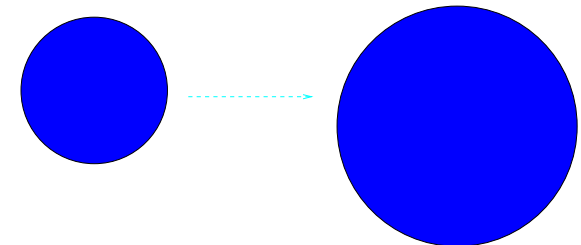
Weak Lensing



Gravitational Lenses

HST · WFPC2

PRC95-43 · ST ScI OPO · October 18, 1995 · K. Ratnatunga (JHU), NASA





- Strong Lensing: Advertising → Delivering



- **Strong Lensing:** Advertising → Delivering

Ad: *“There are data for probing DM , revealing $\langle M \rangle$ of distant galaxies, and resolving R , the internal structure of quasars.”*

Nd: *There are not enough **data**, and most **analyses** are based on the static assumptions.*

(Nd= non delivery, $\langle M \rangle = \bar{M}$ in statistics)



- **Strong Lensing:** Advertising → Delivering

Ad: “There are data for probing DM , revealing $\langle M \rangle$ of distant galaxies, and resolving R , the internal structure of quasars.”

Nd: There are not enough *data*, and most *analyses* are based on the static assumptions.

(Nd= non delivery, $\langle M \rangle = \bar{M}$ in statistics)

- **Weak Lensing:** regular/Gaussian → irregular/Non-Gaussian



- **Strong Lensing:** Advertising → Delivering

Ad: *“There are data for probing DM , revealing $\langle M \rangle$ of distant galaxies, and resolving R , the internal structure of quasars.”*

Nd: *There are not enough **data**, and most **analyses** are based on the static assumptions.*

(Nd= non delivery, $\langle M \rangle = \bar{M}$ in statistics)

- **Weak Lensing:** regular/Gaussian → irregular/Non-Gaussian

How to measure *distortion when shapes of galaxies are irregular*

How to extract *inf given the intervening mass dist. is non Gaussian*



- **Strong Lensing:** Advertising → Delivering

Ad: *“There are data for probing DM , revealing $\langle M \rangle$ of distant galaxies, and resolving R , the internal structure of quasars.”*

Nd: *There are not enough **data**, and most **analyses** are based on the static assumptions.*

(Nd= non delivery, $\langle M \rangle = \bar{M}$ in statistics)

- **Weak Lensing:** regular/Gaussian → irregular/Non-Gaussian

How to measure *distortion when shapes of galaxies are irregular*

How to extract *inf given the intervening mass dist. is non Gaussian*

- **Mine:** Similar



- **Strong Lensing:** Advertising → Delivering

Ad: *“There are data for probing DM , revealing $\langle M \rangle$ of distant galaxies, and resolving R , the internal structure of quasars.”*

Nd: *There are not enough **data**, and most **analyses** are based on the static assumptions.*

(Nd= non delivery, $\langle M \rangle = \bar{M}$ in statistics)

- **Weak Lensing:** regular/Gaussian → irregular/Non-Gaussian

How to measure *distortion when shapes of galaxies are irregular*

How to extract *inf given the intervening mass dist. is non Gaussian*

- **Mine:** Similar

AN: GR, AGN, ISM, ..., 😊



Idea: data D \rightarrow $P(D|p)$ using χ^2 statistics
+ prior on p \rightarrow $P(p|D)$

where $p = (\kappa, \kappa_*, \gamma, \langle M \rangle, R_\lambda, v_e)$, κ is mean surface density, κ_* is surface density in stars, and γ is shear.

1. **Generate** random magnification patterns on a range of $(k, k_*, \gamma, \langle M \rangle)$
2. **Convolve** w. selected disk models R_λ
3. **Generate** light curves (v_e and other nuisance parameters)
4. **Compute** the χ^2 value for each light structure based on a **threshold** value to determine $P(p)$
 - What is the threshold?
 - Is the χ^2 statistic really the scaled sum of squares of residuals after a LS fit? If so, it is equivalent to a Gaussian model. Is this Gaussian model justified?



- Model selection vs. hypothesis testing (what are nulls?)
- Hierarchical models (ok) and final verification (data)

5. *Approximate* Bayesian integrals by sampling/MCMC

- Laplace approximation
- Model averaging and biases

● **Random Thoughts and Comments:**

Extrapolation: Fig 2, data

Pages 6, 7, 8, 9

Comments A1-A9



1. How to define shapes of galaxies that are irregular

Gary's solution:

$$\hat{\gamma} = \frac{\langle \mathbf{e} \rangle}{1 - \langle e^2 \rangle / 2}$$

where noiseless $e^2 = e_+^2 + e_x^2$ is unobserved while the bold-faced $\mathbf{e} = (e'_+, e'_x)$ is observed.

Q: What can we do about the unobserved noiseless e^2 ?

A:

- This leads to a measurement error model in statistics and we can do a deconvolution with an error distribution to estimate it. Is the error distribution (PSF) known?
- An alternative solution may be possible.
- In the worst case, we have $\hat{\gamma}_+ / \hat{\gamma}_x = \langle e'_+ \rangle / \langle e'_x \rangle$ which is observable.



Suggestions:

- **Classify patterns** or shapes after a Procrustes analysis (affine transformation or image registration) of the data —> so the pattern will be defined by a shape code and Procrustes parameters
- **Use topology** (Euler characteristics)
- **Use a mixture** of known shapes with some wavelet basis or other orthogonal basis (e.g. eigen functions in a Laguerre expansion)

Q: biases and distribution?

A: perhaps bootstrap ...



2. Characterize the mass-galaxy likelihood $L(m, g)$
Use a mixture of Gaussian distributions
3. Find a feasible analyses scheme
Use the adaptive estimates, PEM by Sun, Liu and Chen
(06)

Conclusion



- have **built a bridge** between statisticians and astronomers: statisticians found the wonder of GL, astronomers found other statistics to try
- **need more collaborations** between two groups
- GL is challenging, but fun and **has a lot of potential** to probe the DM, ...
- get more data and **validate models with data**